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Belur Math, Howrah - 711202

M. Sc ADMISSION TEST - 2024

MATHEMATICS

Date : 23/07/2024 **Full Marks : 40** Time : 12 noon – 1:00 pm

Instructions for the candidates

- Answer all questions.
- Each question has 4 options out of which only one is correct.
- Tick (✓) the correct option on Answer Sheet.
- The tick (✓) must be very clear – if it is smudgy or not clear, no marks will be awarded.
- Each correct answer carries **2 marks** and for each incorrect answer **1 mark** will be deducted.
- Unanswered questions will not be awarded.
- Multiple answers will be considered as wrong answer.
- Calculator is **not** allowed.

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1. The area (in square unit) of the triangle formed by the pair of straight lines $8x^2 + 10xy + 3y^2 + 26x + 16y + 21 = 0$ and x axis is

   (a) 1 **(b)** 0.5 **(c)** 0.25 **(d)** 0.125 .

2. The equation $r \sin\theta = 2$ in spherical polar coordinates represents

   (a) a circle,
   (b) a right circular cylinder,
   (c) a plane,
   (d) a straight line .

3. The orthogonal trajectories of the family of circles $x^2 + y^2 + 2fy + 1 = 0$, where f is a parameter; is

   (a) $x^2 + y^2 = cx + 1$, (where c being a parameter)
   (b) $x^2 + y^2 = cx$, (where c being a parameter)
   (c) $x^2 + y^2 = c$, (where c being a parameter)
   (d) $c(x^2 + y^2) = x + 1$, (where c being a parameter).

4. What is the coordinate of $(1, 2, 3, 4)$ in $\mathbb{R}^4$ with respect to the ordered basis $\{(1, 0, 0, 0), (0, 2, 0, 0), (0, 0, 3, 0), (0, 0, 0, 4)\}$?

   (a) $(1, 0, 0, 0)$, **(b)** $(0, 1, 0, 0)$, **(c)** $(0, 0, 0, 1)$, **(d)** $(1, 1, 1, 1)$

5. Nullity of the linear operator represented by the matrix $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 6 & 1 \\ -1 & 13 & 0 \end{bmatrix}$ is

   (a) 3 **(b)** 2 **(c)** 1 **(d)** 0 .

6. How many linear transformations are there from $\mathbb{R}^3$ to $\mathbb{R}^2$ which sends $(1, -1, 1)$ to $(1, 0)$, $(1, 1, 1)$ to $(0, 1)$ and $(1, 0, 0)$ to $(1, 1)$?

   (a) infinitely many **(b)** none **(c)** 2 **(d)** 1 .

7. Which of the following statements is true for the degree M of the Lagrange's interpolating polynomial for a data set containing n data points?

   (a) M and n are always equal.
   (b) It is possible, in some cases, to have $M < n$.
   (c) It is possible, in some cases, to have $M > n$.
   (d) No equality or inequality relation exists between M and n.

8. Which of the following statements is correct for the Newton Raphson method for solving equations of the form $f(x) = 0$ in an interval $[a, b]$?

   (a) It is a fixed point method.
   (b) Convergence is guaranteed whenever f is twice differentiable.
   (c) Convergence is guaranteed whenever there is a unique root in the interval $[a, b]$.
   (d) Convergence is guaranteed unconditionally.

9. A particle P possesses two constant velocities u and v, such that u is always parallel to a fixed direction OX and v is always perpendicular to the radius vector OP. The path of the particle is a conic of eccentricity
   (a) $\frac{u}{v}$ **(b)** $\frac{v}{u}$ **(c)** $\frac{u^2}{v}$ **(d)** $\frac{v^2}{u}$

10. The envelope of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters are connected by the relation $ab = c$ is
    (a) $xy = \frac{4}{c^2}$ **(b)** $x^2 = 4cy$ **(c)** $\sqrt{x} + \sqrt{y} = \sqrt{c}$ **(d)** $xy = \frac{c^2}{4}$

11. In which of the following sets, is the curve $y = e^x(\cos x + \sin x),\ x \in (0, 2\pi)$ concave upwards?

    (a) $(0, \frac{\pi}{2}) \cup (\frac{5\pi}{4}, 2\pi)$ **(b)** $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$ **(c)** $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{4}, 2\pi)$ **(d)** $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, 2\pi)$

12. The value of the integral $\int_C \frac{dz}{z^2}$, where C is the positively oriented circle $z = 2e^{i\theta}$ $(-\pi < \theta \leq \pi)$ about the origin is:
    (a) 1 **(b)** 2 **(c)** -1 **(d)** 0

13. $f : \mathbb{R}^2 \to \mathbb{R}$ is defined as $f(x, y) = x^2 - y^2$. Which of the following statements is not true?

    (a) $f(x, 0)$ has a minimum at (0,0).
    (b) $f(0, y)$ has a maximum at (0,0).
    (c) $f(x, y)$ has a saddle point at (0,0).
    (d) Hessian of f is positive definite at (0,0).

14. Consider the statements:
    (A): Every Riemann integrable function defined on $[0, 1]$, must have a primitive on $[0, 1]$.
    (B): Every function having a primitive on $[0, 1]$, must be Riemann integrable on $[0, 1]$.
    Which of the following statements is true?

    (a) Both of (A) and (B) are true.
    (b) (A) is true and (B) is false.
    (c) (B) is true and (A) is false.
    (d) Both of (A) and (B) are false.

15. Which of the following series is conditionally convergent ?

(a) $\sum_{n=1}^{\infty}(-1)^n$

(b) $\sum_{n=1}^{\infty}\frac{1}{n}$

(c) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

(d) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$

16. Let $f:\mathbb{R}\to\mathbb{R}$ satisfies $|f(x)-f(y)-x+y|\leq\sin(|x-y|^2)$ for all $x,y\in\mathbb{R}$. Then f is

(a) differentiable on $\mathbb{R}$ with bounded derivative.

(b) Lipschitz, but not necessarily differentiable with bounded derivative.

(c) uniformly continuous, but not necessarily Lipscitz.

(d) continuous, but not necessarily uniformly continuous.

17. The function $d:\mathbb{R}^2\times\mathbb{R}^2\to[0,\infty)$ is not a metric on $\mathbb{R}^2$, when

(a) $d((x_1,y_1),(x_2,y_2))=\max\{|x_1-x_2|,|y_1-y_2|\}$.

(b) $d((x_1,y_1),(x_2,y_2))=\min\{|x_1-x_2|,|y_1-y_2|\}$.

(c) $d((x_1,y_1),(x_2,y_2))=\left(|x_1-x_2|^2+|y_1-y_2|^2\right)^{1/2}$.

(d) $d((x_1,y_1),(x_2,y_2))=|x_1-x_2|+|y_1-y_2|$.

18. Suppose G is an infinite cyclic group, then G has

(a) only one generator

(b) exactly two generators

(c) more than two but finitely many generators

(d) infinitely many generators.

19. Let S_3 be the symmetric group of all permutations on a set having 3 elements, then the center of S_3 has

(a) 1 element **(b)** 2 elements **(c)** 3 elements **(d)** 6 elements

20. In the ring of all integers, which of the following options is correct

(a) there is no maximal ideal

(b) there is only one maximal ideal

(c) there are exactly two maximal ideals

(d) there are infinitely many maximal ideal.

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